Exponential Decay Current Synapses

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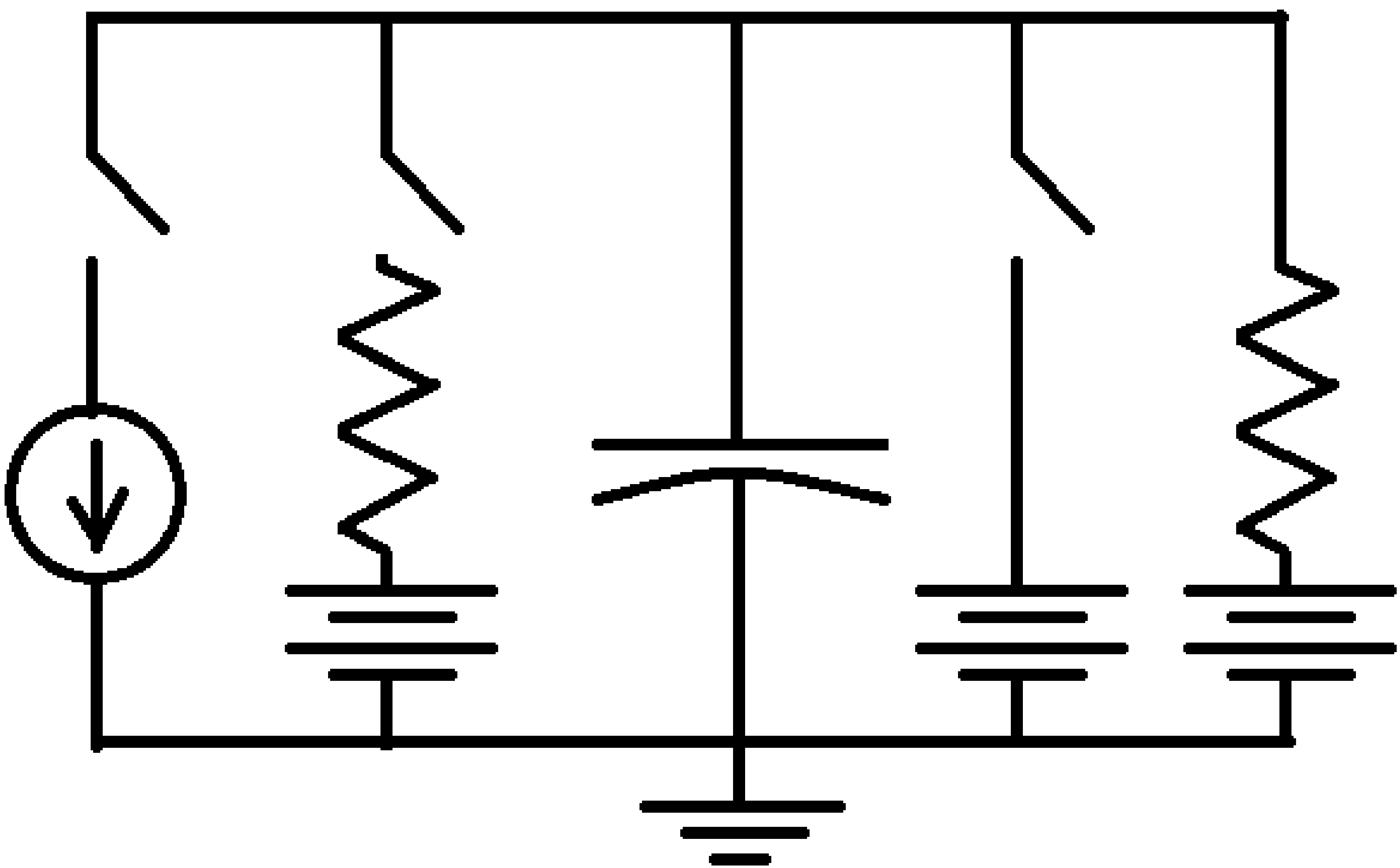
2011 Nov 04 Fri

Overview

- Conductance synapses can be modeled as exponential decay current synapses
- The membrane voltage has an alpha function-like response to an exponential decay current injection
- Using exponential decay current synapses permits a closed-form solution to the membrane voltage in response to a pre-synaptic spike
- When spike times can be solved analytically, fast discrete event simulations can be used

Leaky Integrate-and-Fire



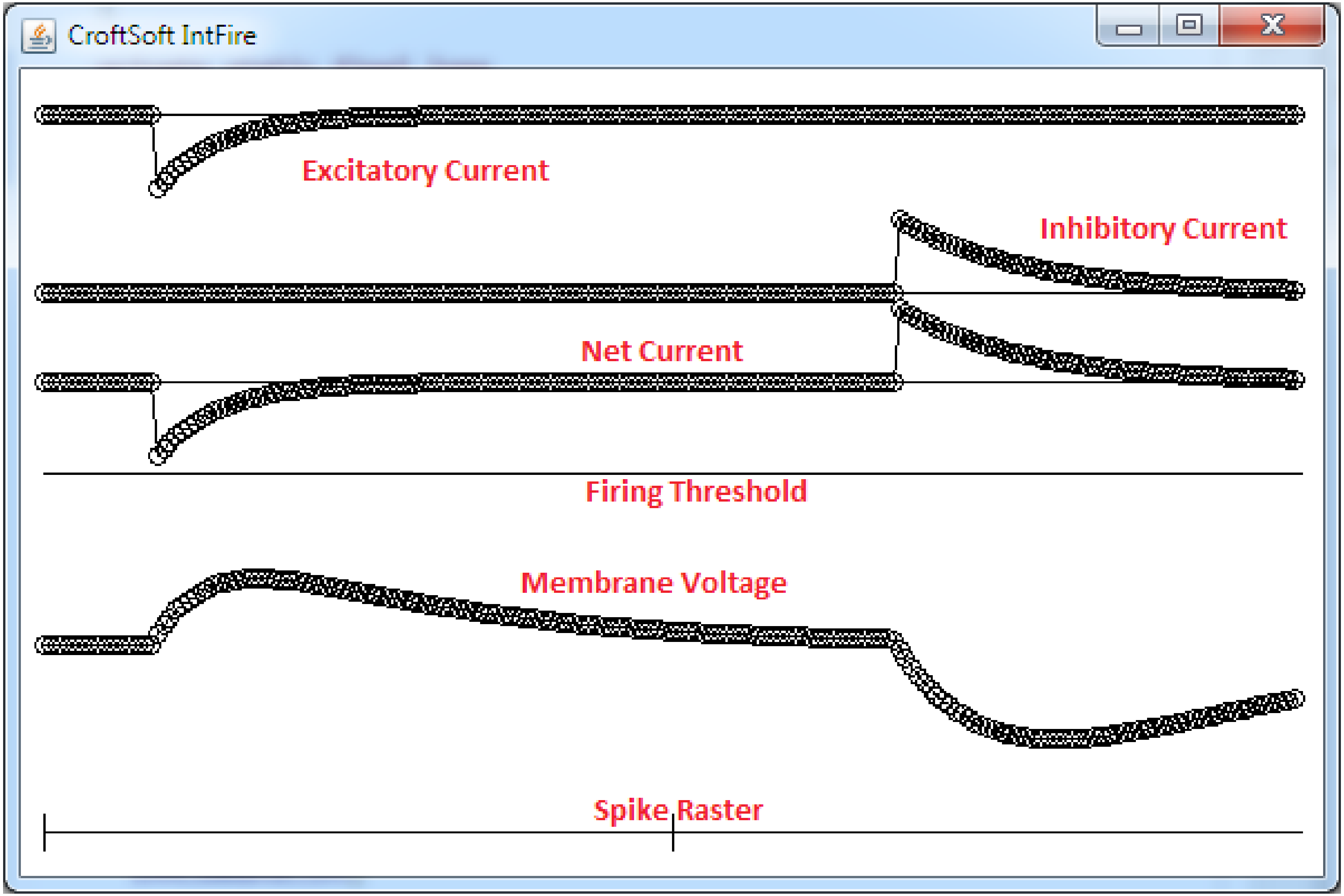
Synapse Types

- Conductance synapses
  - Current depends on difference between membrane voltage and channel reversal potential
  - Nonlinear so no solution to differential equations
  - Requires continuous simulation
- Current synapses
  - Models conductance synapses as injected current
  - Linear injections have solution which can be used to calculate whether voltage exceeds threshold
  - Permits use of discrete event simulation

Simulation Demonstration

- Animated Interactive Simulation Java Applet
- CroftSoft IntFire v1.1
  http://www.CroftSoft.com/library/software/intfire/
- Left-click for excitatory input
- Right-click for inhibitory input

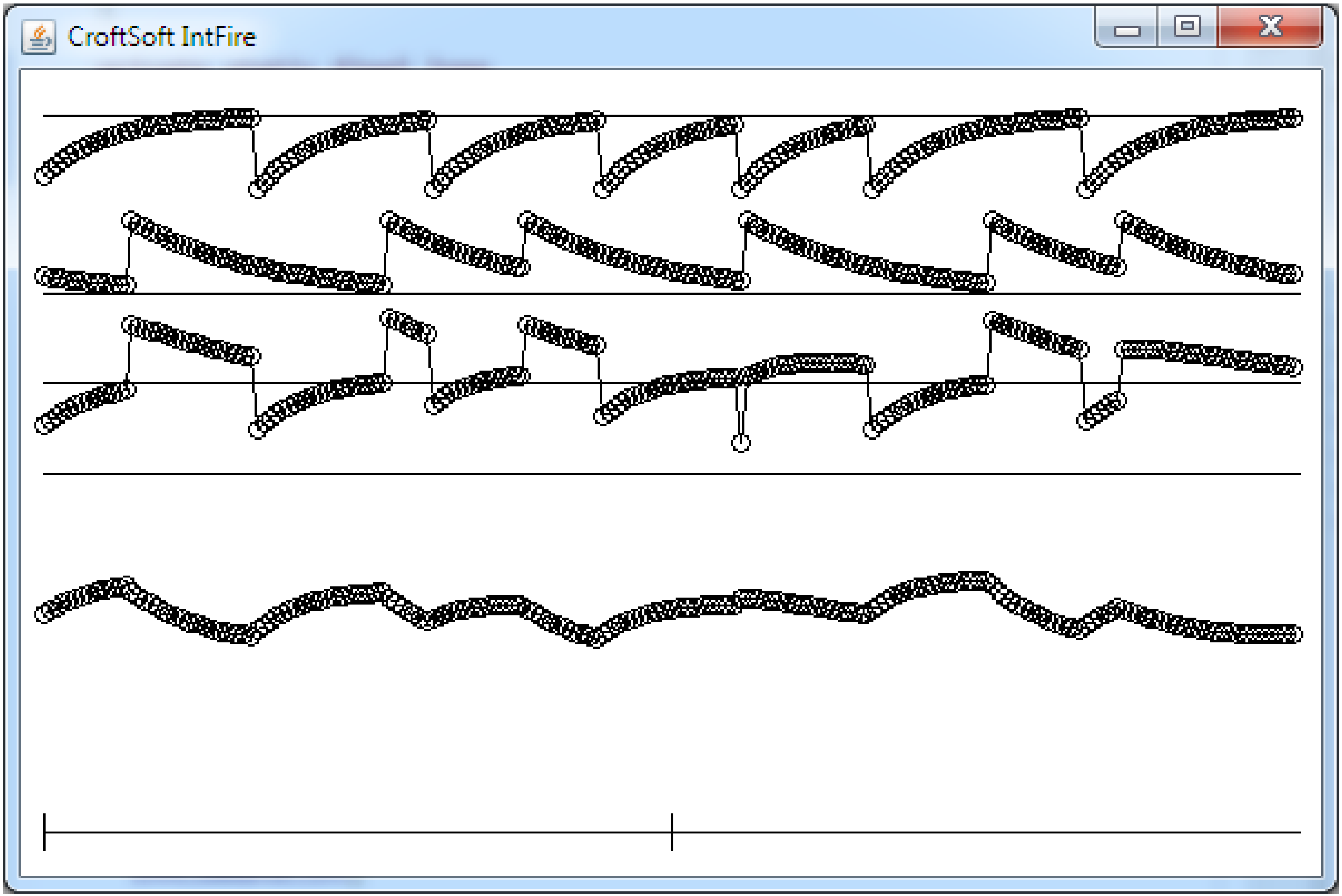
CroftSoft IntFire v1.1



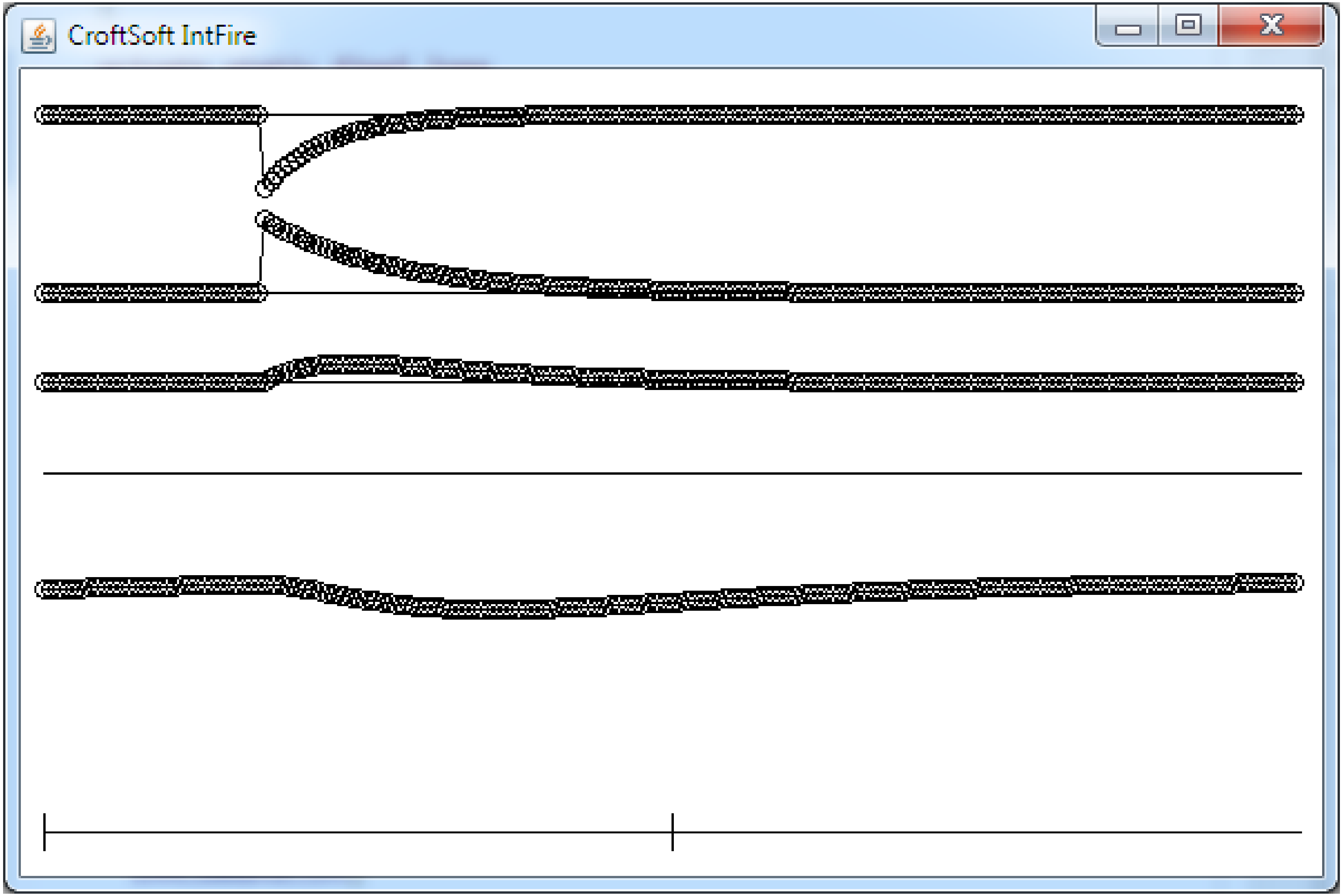
CroftSoft IntFire v1.1

- Modified to show time series data for currents
- Modified to use exponential decay current synapses (injectors)
- Excitatory current negative since it flows into the neuron
- Net current is difference between excitatory current and inhibitory current
- Membrane voltage shows alpha function-like response to exponential decay current input

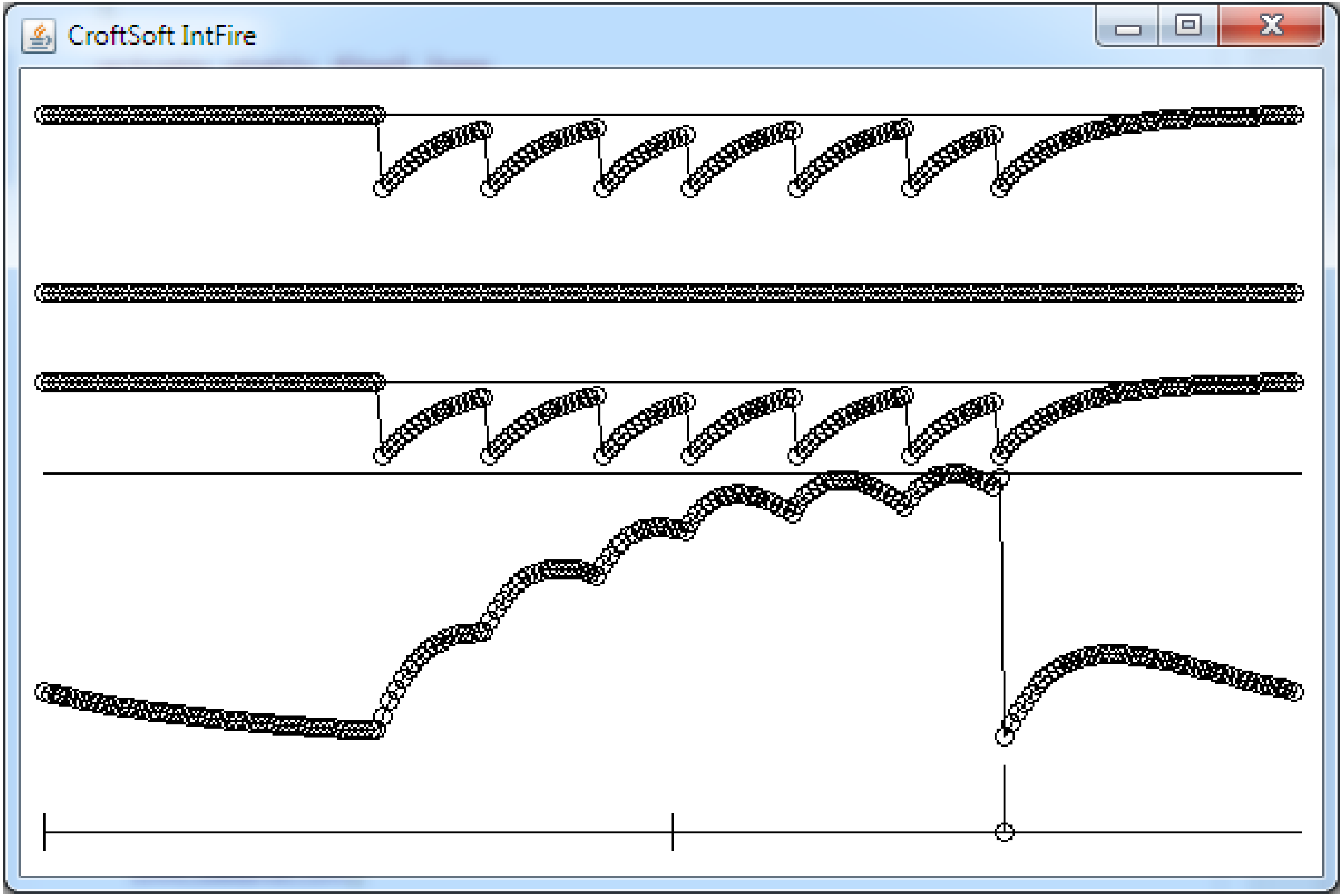
Net Current



Different Decay Rates



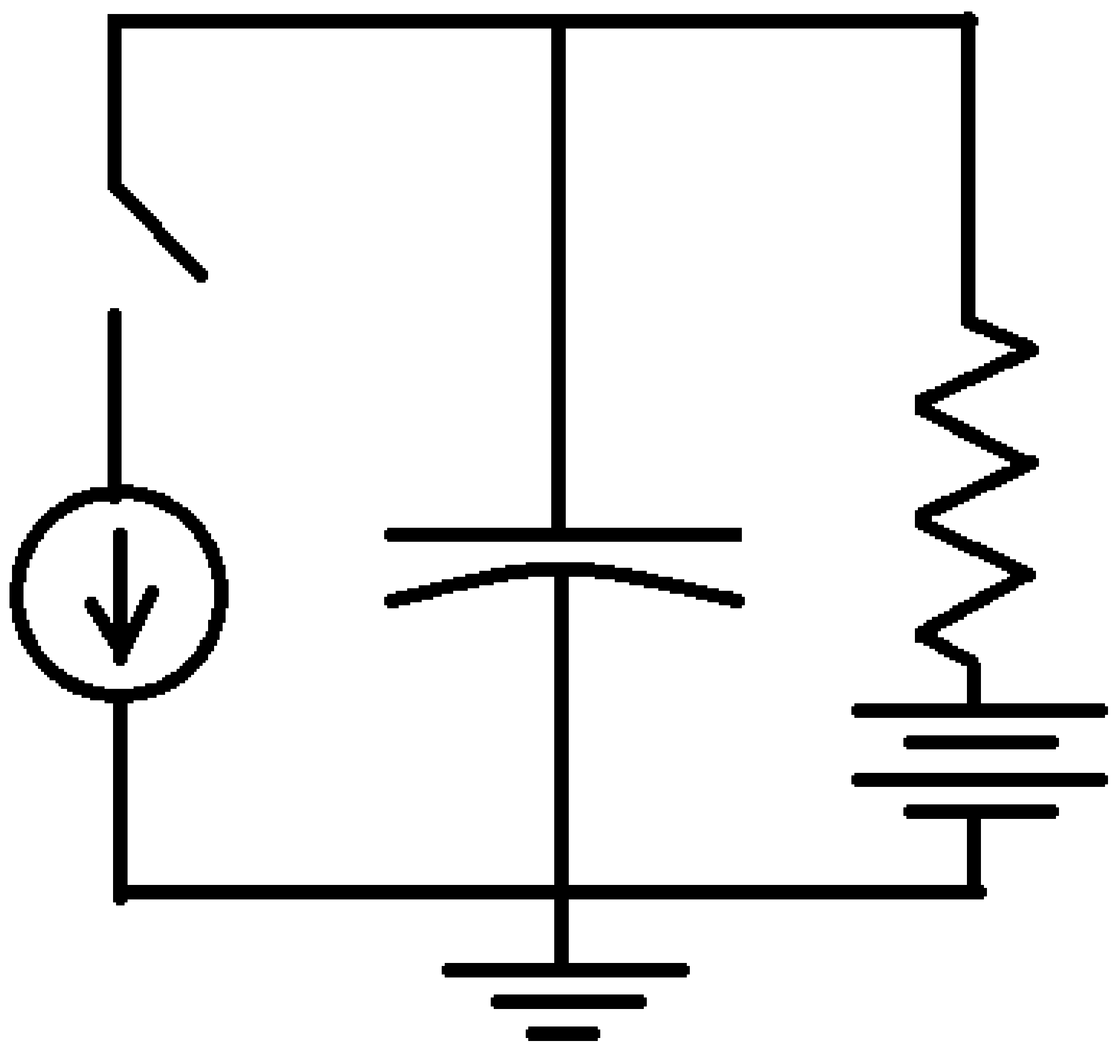
Firing Threshold



Closed-Form Solution

- Continuous simulation updated in small steps
- Discrete event simulation only updated at spike times so simulations can potentially run faster
- Closed-form solution to membrane voltage permits calculation of next spike time in reponse to most recent pre-synaptic spike
- Mihalas and Niebur (2009) use exponential decay current synapses since linear differential equations are analytically solvable

Leaky Integrator



Injected Current

- I injected current
- I' first derivative of the injected current
- I_0 peak current at start of injection
- λ injected current exponential decay rate
- t time since current injected
- $I' = -\lambda \bullet I$ exponential decay differential
- $I = I_0 \bullet e^{-\lambda \bullet t}$ exponential decay closed-form

Capacitive Current

- V membrane voltage at time t
- V' first derivative of the membrane voltage
- V_0 membrane voltage at start of injection
- C membrane capacitance
- I_C capacitive current
- $I_C = C \cdot V'$

Leakage Current

- G leakage conductance
- E leakage conducance reversal potential
- I_L leakage current
- $I_L = G \cdot (V - E)$

Differential Equations

$$I' = -\lambda \cdot I$$

$$I_C = C \cdot V'$$

$$I_L = G \cdot (V - E)$$

$$I_C + I_L + I = 0$$

$$C \cdot V' + G \cdot (V - E) + I_0 \cdot e^{-\lambda \cdot t} = 0$$

Solving Homogeneous

$$a_1 \cdot x' + a_0 \cdot x = F$$

$$r = a_0 / a_1$$

$$q = F / a_1$$

$$x' + r \cdot x = q$$

$$x' + r \cdot x = 0$$

$$x' = -r \cdot x$$

$$h = e^{-\int r}$$

$$x = k \cdot h$$

Standard Form

$$C \cdot V' + G \cdot (V - E) + I_0 \cdot e^{-\lambda \cdot t} = 0$$

$$C \cdot V' + G \cdot (V - E) = -I_0 \cdot e^{-\lambda \cdot t}$$

$$C \cdot V' + G \cdot V - G \cdot E = -I_0 \cdot e^{-\lambda \cdot t}$$

$$C \cdot V' + G \cdot V = G \cdot E - I_0 \cdot e^{-\lambda \cdot t}$$

$$V' + \frac{G}{C} \cdot V = \frac{G}{C} \cdot E - \frac{I_0}{C} \cdot e^{-\lambda \cdot t}$$

Homogeneous Solution (1 of 2)

$$V' + \frac{G}{C} \cdot V = \frac{G}{C} \cdot E - \frac{I_0}{C} \cdot e^{-\lambda \cdot t}$$

$$V' + \frac{G}{C} \cdot V = 0$$

$$V' = -\frac{G}{C} \cdot V$$

$$\frac{V'}{V} = -\frac{G}{C}$$

Homogeneous Solution (2 of 2)

$$\int_0^t \frac{V'}{V} = -\int_0^t \frac{G}{C}$$

$$\ln V = -\frac{G}{C} \cdot t + \alpha$$

$$V = e^{-\frac{G}{C} \cdot t + \alpha}$$

$$V = e^{\alpha} \cdot e^{-\frac{G}{C} \cdot t}$$

$$V = V_0 \cdot e^{-\frac{G}{C} \cdot t}$$

Solving Nonhomogeneous

$$x' + r \cdot x = q$$

$$x = k \cdot h$$

$$x' = k' \cdot h + k \cdot h'$$

$$x' + r \cdot x = \left(k' \cdot h + k \cdot h'\right) + r \cdot \left(k \cdot h\right)$$

$$x' + r \cdot x = k' \cdot h + k \cdot \left(h' + r \cdot h\right)$$

$$h' + r \cdot h = 0$$

$$x' + r \cdot x = k' \cdot h = q$$

$$k' = q / h$$

$$k = \int q / h$$

$$x = k \cdot h$$

Nonhomogeneous Solution (1 of 6)

$$k' = q/h$$

$$k' = \left[\frac{G}{C}\cdot E - \frac{I_0}{C}\cdot e^{-\lambda\cdot t}\right] / \left[V_0\cdot e^{-\frac{G}{C}\cdot t}\right]$$

$$k' = \frac{1}{V_0}\cdot e^{\frac{G}{C}\cdot t}\cdot\left[\frac{G}{C}\cdot E - \frac{I_0}{C}\cdot e^{-\lambda\cdot t}\right]$$

$$k' = \frac{1}{V_0\cdot C}\cdot\left[G\cdot E\cdot e^{\frac{G}{C}\cdot t} - I_0\cdot e^{\left[\frac{G}{C}-\lambda\right]\cdot t}\right]$$

Nonhomogeneous Solution (2 of 6)

$$\int k' = \frac{1}{V_0 \cdot C} \cdot \left[G \cdot E \cdot \int e^{\frac{G}{C} \cdot t} - I_0 \cdot \int e^{\left[\frac{G}{C} - \lambda\right] \cdot t} \right]$$

$$k = \frac{1}{V_0 \cdot C} \cdot \left[G \cdot E \cdot \int e^{\frac{G}{C} \cdot t} - I_0 \cdot \int e^{\left[\frac{G}{C} - \lambda\right] \cdot t} \right] + \alpha$$

$$k = \frac{1}{V_0} \cdot \left[E \cdot e^{\frac{G}{C} \cdot t} - \frac{I_0}{G - C \cdot \lambda} \cdot e^{\left[\frac{G}{C} - \lambda\right] \cdot t} \right] + \alpha$$

Nonhomogeneous Solution (3 of 6)

$$x = k \cdot h$$

$$k = \frac{1}{V_0} \cdot \left[E \cdot e^{\frac{G}{C} \cdot t} - \frac{I_0}{G - C \cdot \lambda} \cdot e^{\left[\frac{G}{C} - \lambda\right] \cdot t} \right] + \alpha$$

$$h = V_0 \cdot e^{-\frac{G}{C} \cdot t}$$

$$V = E - \frac{I_0}{G - C \cdot \lambda} \cdot e^{-\lambda \cdot t} + \alpha \cdot V_0 \cdot e^{-\frac{G}{C} \cdot t}$$

Nonhomogeneous Solution (4 of 6)

$$V = E - \frac{I_0}{G - C \cdot \lambda} \cdot e^{-\lambda \cdot t} + \alpha \cdot V_0 \cdot e^{-\frac{G}{C} \cdot t}$$

$$\alpha \cdot V_0 \cdot e^{-\frac{G}{C} \cdot t} = V - E + \frac{I_0}{G - C \cdot \lambda} \cdot e^{-\lambda \cdot t}$$

$$\alpha = \frac{1}{V_0} \cdot e^{\frac{G}{C} \cdot t} \cdot \left[V - E + \frac{I_0}{G - C \cdot \lambda} \cdot e^{-\lambda \cdot t} \right]$$

Nonhomogeneous Solution (5 of 6)

$$\alpha = \frac{1}{V_0} \cdot e^{\frac{G}{C} \cdot t} \cdot \left[V - E + \frac{I_0}{G - C \cdot \lambda} \cdot e^{-\lambda \cdot t} \right]$$

$$At\ time\ t = 0, V = V_0$$

$$\alpha = \frac{1}{V_0} \cdot 1 \cdot \left[V_0 - E + \frac{I_0}{G - C \cdot \lambda} \cdot 1 \right]$$

$$\alpha = 1 - \frac{E}{V_0} + \frac{I_0}{V_0 \cdot [G - C \cdot \lambda]}$$

Nonhomogeneous Solution (6 of 6)

$$V = E - \frac{I_0}{G - C\cdot\lambda}\cdot e^{-\lambda\cdot t} + \alpha\cdot V_0\cdot e^{-\frac{G}{C}\cdot t}$$

$$\alpha = 1 - \frac{E}{V_0} + \frac{I_0}{V_0\cdot\left[G - C\cdot\lambda\right]}$$

$$V = E + \left[V_0 - E\right]\cdot e^{-\frac{G}{C}\cdot t} + \frac{I_0}{G - C\cdot\lambda}\cdot\left[e^{-\frac{G}{C}\cdot t} - e^{-\lambda\cdot t}\right]$$

References

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